Continuous Distributions

Normal Distribution

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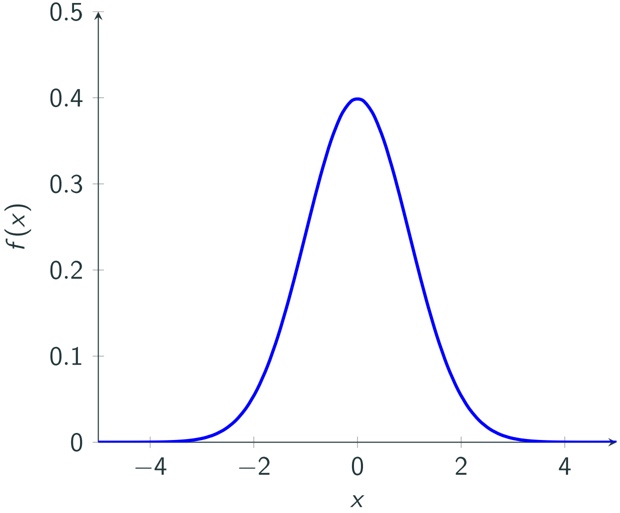
The Standard Normal Distribution

The standard normal distribution is a normal distribution of standardized values called z-scores. A z-score is measured in units of the standard deviation.

The Standard Normal Distribution

The plot with $\mu = 0$ ***and*** $\sigma = 1$



The Standard Normal Distribution

The Z-Score

If X is a normally distributed random variable and $X \sim N(\mu, \sigma)$, then the z-score is:

$$z = \frac{x - \mu}{\sigma}$$

The z-score tells you how many standard deviations the value x is above (to the right of) or below (to the left of) the mean, μ.

The Standard Normal Distribution

Example 1 Suppose $X \sim N(5, 6)$. This says that x is a normally distributed random variable with $\mu = 5$ and $\sigma = 6$. You are given two values of x, $x = 17$ and $x = 1$. Find the z-score.

The Standard Normal Distribution

Example 1 - Solution

When $x = 17: \; z = \dfrac{17 - 5}{6} = 2$

When $x = 1: \; z = \dfrac{1 - 5}{6} = -0.667$

The Standard Normal Distribution

Example 2 The mean height of 15 to 18-year-old males from Chile from 2009 to 2010 was 170 cm with a standard deviation of 6.28 cm. Male heights are known to follow a normal distribution. Let X = the height of a 15 to 18-year-old male from Chile in 2009 to 2010. Then $X \sim N(170, 6.28)$.

The Standard Normal Distribution

Example 2

1. Suppose a 15 to 18-year-old male from Chile was 168 cm tall from 2009 to 2010. What is the z-score?
2. What does this z-score tell us?
3. Suppose that the height of a 15 to 18-year-old male from Chile from 2009 to 2010 has a z-score of $z = 1.27$. What is the male's height?
4. What does this z-score tell us?

The Standard Normal Distribution

Example 2 - Solution

1. $z = \dfrac{168 - 170}{6.28} = -0.32$
2. The height of the male is 0.32 standard deviation to the left of the mean.
3. $x = z \cdot \sigma + \mu = (1.27 \cdot 6.28) + 170 = 178$
4. The height of the male is 1.27 standard deviation to the right of the mean.

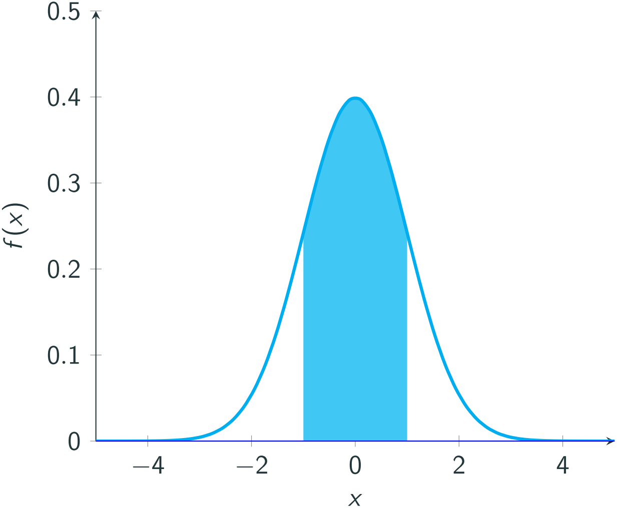
The Standard Normal Distribution

The Empirical Rule

- About 68 percent of the x values lie between $-1 \cdot \sigma$ and $+1 \cdot \sigma$ of the mean, μ (within one standard deviation of the mean).
- About 95 percent of the x values lie between $-2 \cdot \sigma$ and $+2 \cdot \sigma$ of the mean, μ (within two standard deviations of the mean).
- About 99.7 percent of the x values lie between $-3 \cdot \sigma$ and $+3 \cdot \sigma$ of the mean, μ (within three standard deviations of the mean).

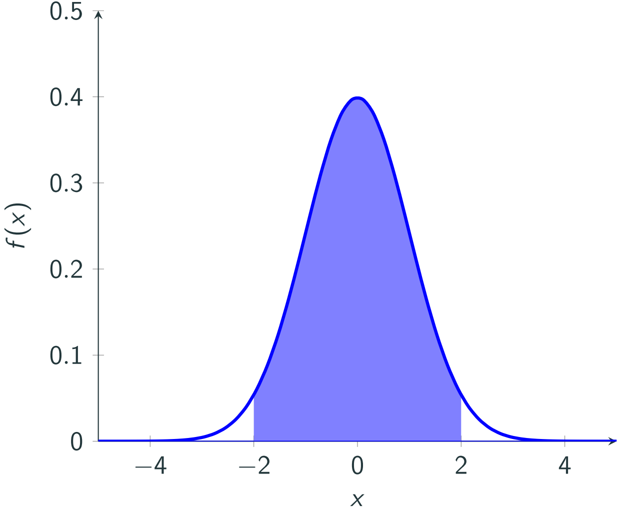
Linear Regression

The Empirical Rule, 68 percent



The Standard Normal Distribution

The Empirical Rule, 95 percent



The Standard Normal Distribution

The Empirical Rule, 99.7 percent